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STABILITY OF AN AXISYMMETRICAL COMPRESSIBLE INVISCID WAKE, (U)  
JUN 78 S Y GERTSENSHTEYN, A V KASHKO

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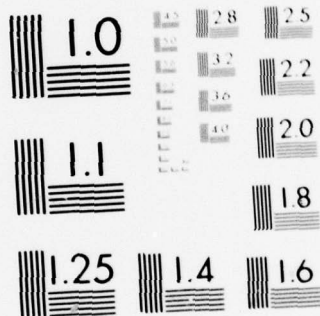
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STABILITY OF AN AXISYMMETRICAL COMPRESSIBLE INVISCID WAKE

By

S. Ya. Gertsenshteyn and A. V. Kashko



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## EDITED TRANSLATION

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By: S. Ya. Gertsenshteyn and A. V. Kashko

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# U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<b><i>А а</i></b>	A, a	Р р	<b><i>Р р</i></b>	R, r
Б б	<b><i>Б б</i></b>	B, b	С с	<b><i>С с</i></b>	S, s
В в	<b><i>В в</i></b>	V, v	Т т	<b><i>Т т</i></b>	T, t
Г г	<b><i>Г г</i></b>	G, g	У у	<b><i>У у</i></b>	U, u
Д д	<b><i>Д д</i></b>	D, d	Ф ф	<b><i>Ф ф</i></b>	F, f
Е е	<b><i>Е е</i></b>	Ye, ye; E, e*	Х х	<b><i>Х х</i></b>	Kh, kh
Ж ж	<b><i>Ж ж</i></b>	Zh, zh	Ц ц	<b><i>Ц ц</i></b>	Ts, ts
З з	<b><i>З з</i></b>	Z, z	Ч ч	<b><i>Ч ч</i></b>	Ch, ch
И и	<b><i>И и</i></b>	I, i	Ш ш	<b><i>Ш ш</i></b>	Sh, sh
Й й	<b><i>Й й</i></b>	Y, y	Щ щ	<b><i>Щ щ</i></b>	Shch, shch
К к	<b><i>К к</i></b>	K, k	Ъ ъ	<b><i>Ъ ъ</i></b>	"
Л л	<b><i>Л л</i></b>	L, l	Ы ы	<b><i>Ы ы</i></b>	Y, y
М м	<b><i>М м</i></b>	M, m	Ь ь	<b><i>Ь ь</i></b>	'
Н н	<b><i>Н н</i></b>	N, n	Э э	<b><i>Э э</i></b>	E, e
О о	<b><i>О о</i></b>	O, o	Ю ю	<b><i>Ю ю</i></b>	Yu, yu
П п	<b><i>П п</i></b>	P, p	Я я	<b><i>Я я</i></b>	Ya, ya

\*ye initially, after vowels, and after ъ, ы; e elsewhere.  
When written as ё in Russian, transliterate as yě or ě.

## RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh <sup>-1</sup>
cos	cos	ch	cosh	arc ch	cosh <sup>-1</sup>
tg	tan	th	tanh	arc th	tanh <sup>-1</sup>
ctg	cot	cth	coth	arc cth	coth <sup>-1</sup>
sec	sec	sch	sech	arc sch	sech <sup>-1</sup>
cosec	csc	csch	csch	arc csch	csch <sup>-1</sup>

Russian	English
rot	curl
lg	log

0932

**STABILITY OF AN AXISYMMETRIC COMPRESSIBLE INVISCID WAKE****S. Ya. Gertsenshteyn and A. V. Kashko****Summary**

This article gives some results of the calculation of flow stability in the wake behind a body flying at a hypersonic speed. The wave numbers, propagation rates and amplification factors characteristic of the oscillations originating in the wake are obtained. The dependences of these values on the Mach number and the temperature gradient on the axis and the periphery of the wake are considered.

The article also gives a short survey of theoretical research on the stability of flow in a distant wake.

1. From many experimental studies [1, 2] it is well-known that sinusoidal oscillations originate in the hypersonic wake behind the "throat" (at a distance of 10-15 diameters of the body). These oscillations develop rather quickly, and flow becomes turbulent at a distance of around 25 diameters from the "throat" below the wake. The study of these oscillations is of special interest, and a number of theoretical investigations deal with this question.

In general, the results were obtained for plane wakes. The purpose of this investigation was to study axisymmetrical compressible wakes.

We will point out that the analogy between plane and axisymmetrical wakes is far from complete. For example, in the axisymmetrical case, it has been shown [11] that two-dimensional growing oscillations do not exist for sufficiently "smoothed" profiles; only a three-dimensional sinusoidal wave can develop in the flow. According to the Squire theorem, in the plane case, plane disturbances are less stable. As the calculations show, the corresponding numerical data for axisymmetrical and plane wakes also



differ considerably from each other.

Obviously, Helmholtz was the first to explain the origination of oscillations in a wake by the instability of the vortex layers forming in the wake. Rayleigh [3] considered and solved the problem of the stability of a vortex sheet. He also solved the problem of the stability of a vortex layer with finite thickness. The problem of the stability of flow in a wake with the distribution of a vortex with finite intensity was first considered by G. I. Petrov [4]. In this report, the velocity profile changed irregularly, having five branches (Fig. 1); the solution was obtained analogously to the above Rayleigh solution. Using the relatively simple calculations in [4], it was shown that, in particular, asymmetrical disturbances develop more intensively than symmetrical. In this same report [4], the generalized Rayleigh method was used to consider the stability of an axisymmetrical wake relative to zonal oscillations. Some of the results of [4] were contradictory, and they were refined in the report by Michalke and Schade [12]. The Rayleigh method was also used in [12], but the velocity profile changed irregularly with more branches than in [4].

We will point out that the overwhelming majority of studies of the stability of wakes concern plane wakes. The results on the stability of axisymmetrical wakes are mainly given in three studies

[11, 4, 13]. As was indicated above, [4] considers the stability of flow relative to zonal oscillations. Some estimates and criteria which are necessary and sufficient for the existence of neutral and growing oscillations are derived in [11] analogously to the known criteria for the plane case. The necessary condition for the existence of growing disturbances obtained in [11] is the most interesting. Below we will briefly explain how this condition can be obtained and what conclusions can easily be drawn from it.

The problem of the stability of an axisymmetrical wake in a perfect incompressible fluid can be reduced to the problem of the proper values for the second-order equation

$$(U-c) \frac{d}{dr} \left\{ \frac{r}{n^2 + \alpha^2 r^2} \frac{d(r\psi)}{dr} \right\} - (U-c) \psi - r \frac{d}{dr} \left( \frac{r\psi'}{n^2 + \alpha^2 r^2} \right) = 0 \quad (1)$$

Here the boundary conditions are the conditions of the limitedness of the solution at  $r=0$  and  $r \rightarrow \infty$ .  $U(r)$  is the velocity profile of the main flow,  $\alpha$  and  $n$  are the assigned parameters (the disturbance was represented as:

$$\psi' = \hat{\psi}(r) \exp(i\alpha z - i\alpha c t + i n \varphi).$$

Multiplying equation (1) by  $\frac{r\bar{\psi}}{U-c}$  (here  $\bar{\psi}$  is the function conjugate to  $\psi$ ), integrating the expression obtained from  $r=0$  to  $r \rightarrow \infty$ , integrating by parts using the known boundary conditions, and separating the real and imaginary part, it is easy to obtain:

$$C_i \int_0^\infty |q|^2 q' dx = 0 \quad (2)$$

Here

$$q(x) = \frac{v}{U-c} G(x); \quad Q = \frac{v v'}{n^2 + c^2 x^2}$$

If  $C_i = 0$ , then it follows from (2) that  $Q'$  should change signs in the interval  $(0, \infty)$ , i.e., at  $C_i \neq 0$ ,  $Q'$  should return to zero at a certain point in interval  $(0, \infty)$ . From this necessary condition, it is not hard to show that  $Q' = 0$  at  $n = 0$  for  $v(x) = U_0 / (1 + \frac{x^2}{x_0^2})^2$ , for example.

Report [11] shows that for this profile, growing disturbances can only exist at  $n = 1$ , i.e., only "sinusoidal" oscillations can be unstable.

Article [11] points out that this conclusion can only be used for "smoothed" velocity profiles.

For profiles with a marked change in velocity, this conclusion is known to be incorrect. The specific computations for growing disturbances are not given in [11].

Study [13] mainly concerns plane wakes in a compressible

inviscid fluid. For the axisymmetrical case, this report gives only isolated calculations for the Gaussian distribution of the velocity profile  $U(r) \cdot P''$  at  $n = 1$ , i.e., for "sinusoidal" waves. Report [13] not only gives specific calculations, but certain necessary criteria for the existence of neutral disturbances and a certain sufficient condition for the existence of growing disturbances are derived. This study is analogous to the studies conducted for an incompressible fluid [5, 6]. The authors also used the studies of Lin<sup>o</sup> and L. Liz of the stability of a compressible boundary layer, which have not been published in the Soviet Union, either.

Besides the above study [13], the stability of a compressible wake was also studied in [7, 14, 15]. The plane case is investigated in [7, 14]. Article [15] considers the plane and axisymmetrical cases. Here the main flow was assumed to be constant in the vicinity of the axis, i.e., the stability of an axisymmetrical vortex sheet in a compressible gas was considered.

2. It is convenient to consider the initial system of equations in the cylindrical coordinate system [8] for studying oscillations in an axisymmetrical wake:



$$\begin{aligned}
\frac{\partial v_1}{\partial t} v_1 + \frac{1}{r} \frac{\partial v_1}{\partial \theta} v_0 + \frac{\partial v_1}{\partial z} v_1 + \frac{\partial v_1}{\partial t} - \frac{1}{r} v_0^2 &= -\frac{1}{\rho} \frac{\partial p}{\partial z} \\
\frac{1}{r} v_1 v_0 + \frac{\partial v_0}{\partial t} v_1 + \frac{1}{r} \frac{\partial v_0}{\partial \theta} v_0 + \frac{\partial v_0}{\partial z} v_1 + \frac{\partial v_0}{\partial t} &= -\frac{1}{r \rho} \frac{\partial p}{\partial \theta} \\
\frac{\partial v_1}{\partial t} v_1 + \frac{1}{r} \frac{\partial v_1}{\partial \theta} v_0 + \frac{\partial v_1}{\partial z} v_1 + \frac{\partial v_1}{\partial t} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} \\
\frac{\partial p}{\partial t} + \frac{\partial(\rho v_1)}{\partial z} + \frac{1}{r} \frac{\partial(\rho v_0')}{\partial \theta} + \frac{\partial(\rho v_2)}{\partial z} + \frac{\rho v_1}{r} &= 0 \\
c_v \rho \frac{dT}{dt} &= A \frac{p}{\rho} \frac{dp}{dt} \\
p &= \rho R T.
\end{aligned} \tag{3}$$

We will represent the unknown solution of the system in the form of the sum of the main flow

$$\begin{aligned}
\bar{v}_{acc.} (v_{acc.} = V_1(z), v_{acc.} = 0, v_{acc.} = 0, \\
T_{acc.} = \bar{T}(z), p_{acc.} = \bar{p}, \rho_{acc.} = \bar{\rho})
\end{aligned}$$

and a certain sufficiently small disturbance:

$$\begin{aligned}
v_1 &= V_1 + v_1', v_2 = v_2', v_0 = v_0', \\
p &= \bar{p} + p', \rho = \bar{\rho} + \rho', \\
T &= \bar{T} + T'.
\end{aligned} \tag{4}$$

Substituting (4) in (3) and disregarding the terms which are quadratic relative to the disturbance components, we will obtain a



system of six linear equations relative to six unknown functions

$$v'_1, v'_2, v'_3, p', T', \rho'$$

$$\frac{\partial v'_1}{\partial t} + V_1 \frac{\partial v'_1}{\partial z} = -\frac{1}{\bar{\rho}} \frac{\partial p'}{\partial z}$$

$$\frac{\partial v'_3}{\partial t} + V_1 \frac{\partial v'_3}{\partial z} = -\frac{1}{\bar{\rho}} \frac{\partial p'}{\partial \theta}$$

$$\frac{\partial v'_2}{\partial t} + v'_1 \frac{\partial V_1}{\partial z} + V_1 \frac{\partial v'_2}{\partial z} = -\frac{1}{\bar{\rho}} \frac{\partial p'}{\partial z}$$

$$\frac{\partial p'}{\partial t} + v'_1 \frac{\partial \bar{p}}{\partial z} + V_1 \frac{\partial p'}{\partial z} + \bar{p} \left[ \frac{\partial v'_1}{\partial z} + \frac{1}{z} \frac{\partial v'_3}{\partial \theta} + \frac{\partial v'_2}{\partial z} + \frac{v'_1}{z} \right] = 0$$

(5)

$$\bar{p} \frac{\partial T'}{\partial t} + \bar{p} \left[ v'_1 \frac{\partial \bar{T}}{\partial z} \right] = (1 - \Gamma_*) \bar{p} \left[ \frac{\partial v'_2}{\partial z} + \frac{\partial v'_1}{\partial z} + \frac{1}{z} \frac{\partial v'_3}{\partial \theta} + \frac{v'_1}{z} \right]$$

$$\rho' = \bar{\rho} T' + \bar{p} T'$$

If we search for all of the disturbance components in the form

$$q' = \hat{q}(\tau) \exp[i\beta t + i\alpha z + i\gamma \varphi]$$

it is easy to obtain the system of ordinary differential equations for determining function  $\hat{q}(\tau)$  from system (5):

$$\begin{aligned}
i\rho \hat{v}_1 + V_1 i\alpha \hat{v}_1 &= -\frac{1}{\rho} \hat{p}' \\
i\rho \hat{v}_2 + V_2 i\alpha \hat{v}_2 &= -\frac{1}{\tau \rho} i\gamma \hat{p} \\
i\rho \hat{v}_1 + \hat{v}_2 \bar{V}_2'(z) + i\alpha \bar{V}_2(z) \hat{v}_2 &= -\frac{1}{\rho} i\alpha \hat{p} \\
i\rho \hat{p} + i\alpha V_1 \hat{p} + \bar{p} \hat{v}_1 + \bar{p} \left[ \hat{v}_1' + \frac{1}{\tau} i\gamma \hat{v}_2 + i\alpha \hat{v}_2 + \frac{\hat{v}_2}{\tau} \right] &= 0 \\
\bar{p} i\rho \hat{T} + \bar{p} \hat{v}_2 \bar{T}'(z) + \bar{p} i\alpha V_1 \hat{T} &= (1-\gamma_0) \bar{p} \left[ i\alpha \hat{v}_1 + \hat{v}_1 + i\frac{\tau}{\tau} \hat{v}_2 + \frac{\hat{v}_2}{\tau} \right] \\
\hat{p} &= \hat{p} \bar{T} + \bar{p} \hat{T}
\end{aligned} \tag{6}$$

We will introduce the dimensionless variables:  $v_i = \frac{v_i'}{u_*}$ ,  $r = \frac{r'}{T_*}$

$$\bar{t} = \frac{u_*}{\alpha} t, \quad P = \frac{P'}{P_0}, \quad \rho = \frac{\rho'}{\rho_0}, \quad \bar{z} = \frac{z}{\alpha}, \quad \tau = \frac{\tau}{\alpha} \tag{7}$$

In dimensionless variables, system (6) has the form:

$$i\rho \hat{v}_1 + V_1 + i\alpha \hat{v}_1 = -\frac{1}{\rho} \frac{1}{M^2 \gamma_0} \hat{p}' \tag{8}$$

$$i\rho \hat{v}_2 + V_2 + i\alpha \hat{v}_2 = -\frac{1}{\tau \rho} i\gamma \hat{p} \frac{1}{M^2 \gamma_0} \tag{9}$$

$$i\rho_1 (\rho + \alpha V_2) + \hat{v}_2 V_2'(z) = -\frac{1}{\rho} \frac{i\alpha \hat{p}}{M^2 \gamma_0} \tag{10}$$

$$i\alpha\hat{p} + i\alpha V_0\hat{p} + \hat{p}\hat{v}_0 + \hat{p}\left(\hat{v}_0 + \frac{1}{\gamma}\gamma\hat{v}_0 + i\alpha\hat{v}_0 + \frac{v_0}{\gamma}\right) = 0 \quad (11)$$

$$\hat{p}i\alpha\hat{T} + \hat{p}\hat{v}_0\hat{T}'_{(v)} + \hat{p}i\alpha V_0\hat{T} = \quad (12)$$

$$= (1 - \gamma_0)\hat{p}\left[i\alpha\hat{v}_0 + v_0' + i\frac{\gamma}{\gamma_0}v_0 + \frac{v_0}{\gamma}\right] \quad (13)$$

$$\hat{p} = \hat{p}\hat{T} + \hat{p}\hat{T} \quad (13)$$

Here  $M$  is the Mach number:  $M^2 = \frac{u_0^2}{\gamma_0 R T_0}$ .

For further calculations, it is convenient to reduce system (8) to one equation. We will write the equation for pressure  $\hat{P}$ . First it is expedient to find the expression for  $\hat{v}_0$  from  $\hat{P}$  and  $\hat{P}^0$ . In order to do this, it suffices to substitute the expression for  $\hat{v}_0$  from equation (8) in equation (10). Substituting the expression for  $\hat{T}$  from equation (13) in equation (12), then expressing it by  $\hat{v}_0, \hat{v}_0'$ , and  $\hat{v}_0$  obtained from (11) instead of  $\hat{p}$  we will obtain the relationship among  $\hat{p}, \hat{p}', \hat{v}_0, \hat{v}_0', \hat{v}_0, \hat{v}_0'$ . If we now use the previously determined expressions for  $\hat{v}_0, \hat{v}_0'$  and  $\hat{v}_0$  by  $\hat{P}$  and  $\hat{P}^0$ , we will obtain the following equation for  $\hat{P}$ :

$$\hat{p}\left\{\frac{a_0}{\gamma} + \frac{\gamma}{\gamma_0 M^2} + \frac{a_0^2}{\gamma_0 M^2}\right\} + \left[-\frac{1}{\alpha_0 \hat{p}} \hat{p}' \frac{1}{\gamma_0 M^2}\right] \frac{1}{\hat{p}} \gamma_0 + \left[-\frac{1}{\alpha_0 \hat{p}} \hat{p}' \frac{1}{\gamma_0 M^2}\right] \left\{\gamma_0 \hat{p} \left[-\frac{i\alpha}{\alpha_0} V_0 + \frac{1}{\gamma}\right]\right\} = 0. \quad (14)$$

Here

$$\alpha = i[\beta + \alpha V_1]$$

3. It is easy to substitute the boundary conditions for equation (14):  $P(0) = 0$  at  $x=0$ , and at  $x \rightarrow \infty$ , the absolute value of solution  $P(x)$  should be limited. However, it is not advisable to numerically integrate equation (14) near point  $x=0$ , since expressions of the form  $\frac{1}{x}$  and  $\frac{1}{x^2}$  are in the coefficients. Therefore, it is logical to use the asymptotic representation of the solution in the vicinity of zero.

In order to perform the numerical calculation, it is also necessary to replace the boundary conditions at infinity by the boundary condition at a finite distance. This can be done by using known asymptotic expansions. We will give the corresponding computations below.

If  $V_1 = 1$  and  $\bar{T} = 1$  at  $x > 1$ , equation (14) assumes the form:

$$\hat{P} \left\{ \frac{1}{i(\beta + \alpha)} \left[ (i\beta + i\alpha)^2 + \frac{r^2}{x^2 M^2} + \frac{\alpha^2}{M^2} \right] \right\} - \hat{P}'' \frac{1}{i(\beta + \alpha)} \frac{1}{M^2} - \frac{1}{i(\beta + \alpha)} \hat{P}' \frac{1}{M^2} \frac{1}{x} = 0. \quad (15)$$

Or



$$\hat{p}'' + \frac{1}{r} \hat{p}' + \hat{p} \left( -\frac{I^2}{r^2} - \alpha^2 + M^2(\alpha, \beta) \right) = 0. \quad (16)$$

The asymptotic behavior of the limited solution to equation (16) at infinity is well-known [9, 10]:

$$p(r) \approx \text{const} \frac{1}{r^2} e^{-\lambda r}, \quad (17)$$

where

$$\lambda^2 + M^2(\alpha, \beta) - \alpha^2 = 0 \quad (18)$$

and

$$\text{Re } \lambda > 0$$

We are not considering the case  $\text{Re } \lambda = 0$ , since the boundary condition at infinity is satisfied automatically for any  $\beta$  at

$M^2(\alpha, \beta) - \alpha^2 < 0$  (the case of a continuous spectrum). Using (17), it is easy to see that at sufficiently large values of  $r$ , the following relationship should be satisfied with great accuracy:

$$p + \frac{1}{2} \frac{1}{r} p + \lambda p = 0. \quad (19)$$

This relationship (19) will also replace the boundary condition at infinity in the future.



We will write the asymptotic solution to equation (14) near zero. It is convenient to rewrite equation (14) as follows:

$$\rho' \cdot \left[ \frac{\beta a_1'}{\beta a_1} + \left[ \frac{a_1'}{a_1} - \frac{1}{\gamma} \right] \right] \hat{\rho}' + \left[ \frac{a_1' M^2}{\gamma} + \frac{\gamma^2 + a_1'^2}{\gamma^3} \right] \hat{\rho} \quad (20)$$

We will represent equation (20) in the form of a series:

$$\rho = \gamma^{\lambda} \sum_{n=0}^{\infty} C_n \gamma^n$$

Gathering the terms of the form  $\gamma^{\lambda+n}$ , we will have:

$$\lambda(\lambda+1)C_0 = \lambda(-C_0) + \gamma^2 C_0$$

Hence:

$$\lambda^2 = \gamma^2$$

It is easy to find the remaining coefficients of the series from  $C_0$ . For example, by gathering the terms of the form  $\gamma^{\lambda+1}$ , we will obtain  $C_1$ :

$$C_1 \cdot \lambda \cdot (\lambda+1) - (\lambda+1) \cdot C_1 + \gamma^2 C_1 + \lambda C_0 \left[ \frac{(\beta a_1')}{\beta a_1} \Big|_{\gamma=0} + \frac{a_1'}{a_1} \Big|_{\gamma=0} \right]$$

Thus, the unknown solution behaves like  $\gamma^{\lambda}$  near zero. Here it is assumed that  $\gamma \neq 0$ . For  $\gamma = 0$ , we can also obtain the corresponding asymptotic representation analogously to the above procedure [27].

4. These calculations made it possible to get an idea of the nature of the oscillations in a supersonic wake. As an example, we considered flow in the wake behind a body in the range of Mach numbers from 10 to 30. The flow instability is considered in the wake behind the "throat," at a distance of 10-15 times the size of the body. The data on the velocity and temperature distributions were taken from [1, 2, 13]:

$$\begin{aligned}
 U &= 1 - \Delta u e^{-\xi^2}, \\
 \bar{T} &= 1 + \Delta T e^{-\xi^2}, \\
 \xi &= \int_0^x \frac{dx}{T(x)}, \\
 \Delta u &\approx 1 - f \approx 1 - (1 - M_\infty^{-1/2})^{1/2}, \\
 \frac{h_p(0)}{h_\infty} &\approx (1 - f^2)(T - 1) \frac{M_\infty^2}{2}.
 \end{aligned}$$

Here  $U(x)$  is the dimensionless velocity,  $\bar{T}(x)$  is the temperature distribution,  $h_p(x)$  is the value of the enthalpy in the cross section we selected on the axis of the wake, and  $h_\infty$  is the value of the enthalpy in the incoming flow.

The results of the calculations are partially shown in graphs

1-6. As Figures 1, 2, 3 and 4 show, the amplification factor decreases with the increase in the Mach number  $M$ , but extremely slightly. Therefore, the well-known marked increase in flow stability with the increase in the Mach number can most probably be explained by the high phase velocity of the propagation of the disturbance in the compressible wake (Fig. 5) - (the disturbances are pulled into the flow very quickly, without being able to develop).

Fig. 1.

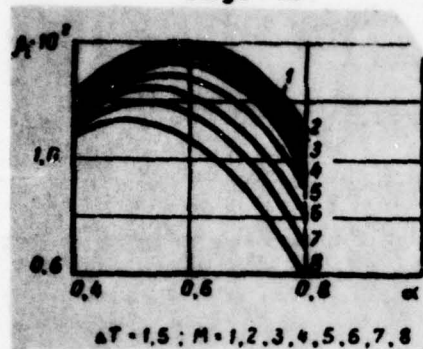


Fig. 2.

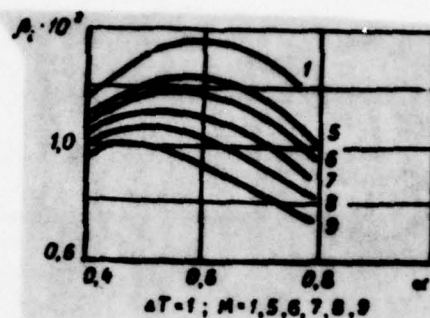


Fig. 3.

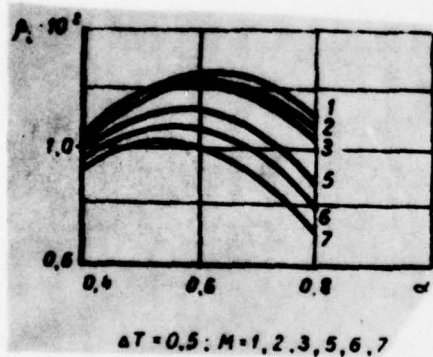
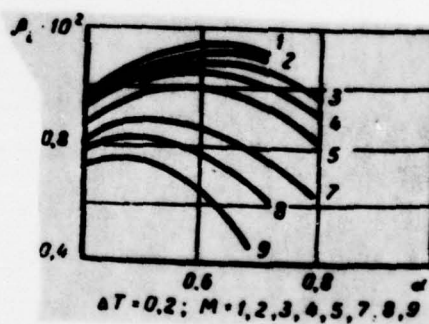
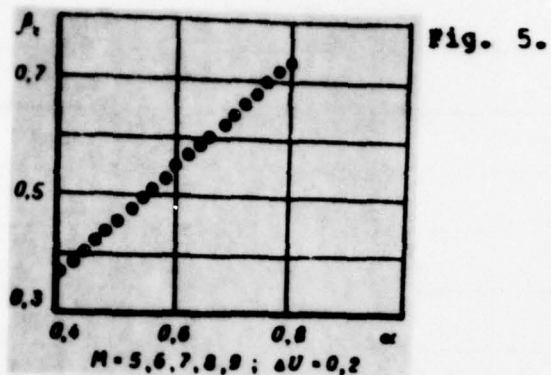


Fig. 4.





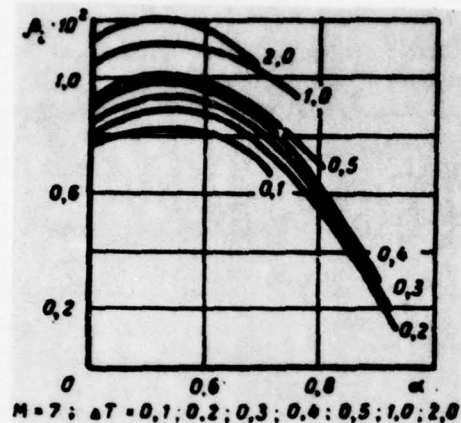


It is interesting to compare the results obtained with the known results [13] of the stability of flat jets. The comparison shows that the amplification factors for flat jets are approximately four times higher than for axisymmetrical jets.

From these calculations we can also see that the extreme wave number  $\alpha^*$ , which corresponds to the maximum amplification factor, decreases with the increase in the Mach number, while the phase velocity  $c^*$  remains virtually unchanged. We will also point out that the phase velocity varies little with the change in the wave number  $\alpha$  (at a fixed Mach number) - Fig. 5.



Fig. 6.



Separate calculations for computing the effects of a hot "core" for an axisymmetrical wake were made in this study.

The amplification factor  $\alpha c_1$  increases approximately 1.5 times (Fig. 6) when  $\Delta T$  increased (from 0.1 to 2.0), while the extreme wave number  $\alpha^*$  decreased by approximately 1/10 of its initial value when  $\Delta T$  increased from 0.1 to 2.0. The rate of propagation of the disturbance in the range of change in question changes very insignificantly. We will point out that for observers moving at the rate of propagation of the disturbance  $c_1 \approx \frac{A_1}{\alpha}$ , the local velocity of the flow whose stability is being investigated is lower than the local speed of sound. This situation holds throughout the range of change in  $\Delta T$  and  $M$  in question.

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